

# A Services-Led Growth Escalator?

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## Abstract

Premature deindustrialisation suggests that services will play an increasingly pivotal role in driving the long-run growth trajectory of developing countries. This paper documents three new facts on productivity growth in services. First, the variance of services productivity growth is higher in developing countries than advanced ones. Second, services productivity growth (or lack thereof) is driven by broad-based improvements (deteriorations) across the different services sub-sectors ('within' channel) as opposed to sub-sector reallocations of employment to more productive services industries ('between' channel). Third, the 'within' channel's effect is partly driven by conditional convergence in services productivity between countries. Combined, these facts motivate a theoretical framework where, like the one-sector neoclassical growth model, the social planner faces a standard consumption-investment trade-off. Differently from the benchmark model however, the planner faces an additional static problem which involves allocating investment between two competing technologies. The first is a catch-up technology while the second is a CRS technology. The gains of the former depend on the level of development that is, the further away from the frontier the higher the returns. Crucially, a country-specific growth efficiency parameter interacts solely with the catch-up technology, scaling the marginal gains from this allocation. Productivity growth in the transitional dynamics therefore depends on three factors: the distance to the frontier, the investment rate and the growth efficiency level. Using the World Input-Output Database (WIOD), I calibrate the growth efficiency level to match the 2000-2005 compound annual growth rate of services value added per worker. The calibration outcomes are used for several quantitative exercises, including a variance decomposition of productivity growth in services as well as two counterfactual exercises. A key result of the paper is that assigning sample maximum growth efficiency levels and investment rates raises average growth among developing countries from an observed 3.6% to a counterfactual rate of 18.7%, closing 86.7% of the gap between observed average growth and the maximum potential growth rate.

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# 1 Introduction

Economists have traditionally held the view that the manufacturing sector is fundamental for developing countries' quest to narrow cross-country income differences. Proponents of this view argue that manufacturing can serve as a 'growth escalator' – a view founded upon the sector's unique ability to generate jobs for unskilled labour and absorb the latest technologies relatively quickly from the world technology frontier. Central to this view is the evidence brought to light by [Rodrik \(2013\)](#) who, using UNIDO data from 1965 to 2005, finds evidence of unconditional convergence unique to only the manufacturing sector. The implications of this are stark, and would imply that by solely allowing the forces of structural transformation to materialise – particularly in the early stages when manufacturing begins to take on a more prominent role in the economy – developing countries can catch up to the frontier.

Are these forces strong enough to close the cross-country income gap? The evidence suggests that this is hardly the case. [Johnson & Papageorgiou \(2020\)](#), for instance, find no evidence of aggregate unconditional convergence studying 182 countries between 1960 and 2010 using the Penn World Tables (PWT) 7.1. If one was to reconcile this result with the insights provided by [Rodrik \(2013\)](#), the sensible conclusion would suggest that the manufacturing sector is too small to drive convergence at the aggregate level. Granted, this result has not been taken at face value by some. It is disputed by [Patel et al. \(2021\)](#), for example, who find strong evidence in favour of aggregate unconditional convergence since the mid-1990s across a range of data sources including the World Bank's World Development Indicators, Maddison and the PWT 10.0. Nevertheless, in acknowledging that the rate of aggregate convergence is somewhat slow, it is implied that developing countries should perhaps consider alternative ways to accelerate this pace of convergence.

Two more recent phenomena suggest that the 'manufacturing-as-a-growth escalator' paradigm has been losing even more ground. The first relates to the recent trend of skill-biased technical change. As [Violante \(2008\)](#) documents, technical change is becoming factor-biased in favour of high-skill workers, hence the growing wage premium in advanced economies. For developing countries, this is bad news. In a recent essay, [Rodrik \(2018\)](#) argues that the growing complementarities between technological progress and high-skilled labour in advanced countries will impede the process of technology diffusion and adoption in developing countries. Barring major investments in human capital, the traditional assumption that the manufacturing sector is a sound absorbent of the latest technologies appears to be gradually eroding.

The second phenomenon relates to the finding of premature deindustrialisation. The Kuznets facts posit that over the development process, countries' share of agriculture value added and employment declines with income at the expense of services. The manufacturing sector takes a different path, experiencing a hump whereby it rises at first, peaks and declines as the country develops ([Herrendorf et al. 2014](#)). It is a well-documented fact that the peak of the manufacturing hump that developing countries face today are lower and take place earlier in the development process compared to those experienced by the early industrialisers ([Rodrik 2016](#)). Thus, even if one was to still believe in the unconditional convergence in manufacturing result, premature deindustrialisation would imply that the manufacturing sector is playing a less prominent role in developing economies' growth trajectories at the expense of services. Taken together, these two facts reinforce the view that the traditional path of long-run development taken by early industrialisers may no longer be suitable for today's developing countries.

In light of this, I investigate the potential for services to function as a 'growth escalator' and help narrow global income differences. The canonical model of structural transformation would suggest that services productivity growth is the slowest of the three sectors ([Ngai & Pissarides 2007](#)). This paper therefore studies the sources of productivity growth in services to better understand how countries can avoid an equilibrium dominated by slow-growing services sectors and in fact leverage the inevitable rise of 'service economy' to accelerate growth prospects. I begin by documenting three facts about productivity growth in services using data from the Groningen Growth & Development Centre's (GGDC) 10-sector database between 1975 and 2009 ([Timmer, De Vries & De Vries 2015](#)). First, the variance in services productivity growth is higher in developing countries than advanced ones. Second, performing a productivity growth decomposition à la [McMillan et al. \(2014\)](#) reveals that services productivity growth is primarily driven by the 'within' channel. This corresponds to productivity growth within different sub-sectors inside services as opposed to the 'between' channel which involves the reallocation of labour to more productive services sub-sectors. Third, I find evidence of conditional convergence in the services sector across countries. In other words, countries further away from the services productivity frontier tend to grow faster conditional on countries' time-invariant structural characteristics. These findings are consistent with the lack of evidence in favour of unconditional convergence at the

aggregate level.

I then propose a mechanism that is consistent with these facts. The environmental set-up resembles that of the neoclassical growth model in that the social planner faces the standard consumption-investment trade-off. The planner's objective is to maximise the economy's welfare, and the investment is used to grow the domestic technology which is the only input in the production of a final service. This set-up implicitly ignores any sub-sector reallocation effects and focuses exclusively on the 'within' channel to maintain consistency with the second empirical fact. The novelty of the exercise lies in the introduction of a simultaneous static decision that the planner must make. In the model, investment can either be allocated to a catch-up or constant returns to scale (CRS) technology. The returns of both technologies depend on the level of development. If the economy is far away from the world technology frontier, the catch-up technology generates more favourable returns. As the domestic technology approaches the frontier, the returns from the catch-up technology fall until eventually the CRS technology is preferred. Crucially, only the catch-up technology interacts with a country-services-specific growth efficiency parameter that captures persistent time-invariant structural characteristics. Variation in the services-specific growth efficiency level across countries drives both the variation in productivity growth in services and explains why some countries converge to the frontier and others do not, thus accommodating the first and third empirical facts. Accordingly, the transitional dynamics of the model reveal that there are three sources of productivity growth: the distance to the frontier, the growth efficiency level and the investment rate.

Finally, I calibrate the model using data from the GGDC World Input-Output Database (WIOD) (Timmer, Dietzenbacher, Los, Stehrer & De Vries 2015). The advantage of using the WIOD is that not only can I retrieve services value added, I can also extract the shares of this going to consumption and investment. Using this data, I am able to back out key parameters such as the growth efficiency level and the CRS technology return. The results of the calibration exercise allow me to conduct multiple quantitative exercises including a services productivity growth decomposition as well as two counterfactual exercises. The first assigns counterfactual combinations of investment rates, initial output and growth efficiency levels to different developing countries. The second exercise isolates the impact of varying growth efficiency levels on the trajectory of services productivity level.

A key result of the paper is that assigning sample maximum growth efficiency levels and investment rates raises average growth among developing countries from an observed 3.6% to a counterfactual rate of 18.7%, closing 86.7% of the gap between observed average growth and the maximum potential growth rate. I show that by solely replacing the growth efficiency level of Brazil (the sample minimum) with the growth efficiency of India (the sample maximum), the time to convergence with the frontier is reduced by half. These results indicate that indeed, services cannot serve as a 'growth escalator' in the way that manufacturing used to (given the conditional convergence in services as opposed to unconditional convergence observed in manufacturing). Closing the cross-country income gap will require raising both investment in services and the growth efficiency level of developing countries, though more work needs to be done to understand the components of the growth efficiency level.

Admittedly, this paper is not the first to focus exclusively on services. Early work in this regard include Stigler et al. (1956), Kuznets (1957) and Fuchs et al. (1968). More recently, Buera & Kaboski (2012) attempt to rationalise the simultaneous rise in the share of high-skill services value added and growing wage premia in the US, arguing that this mostly reflects a story of preferences. In their framework, high-skill workers develop a comparative advantage vis-a-vis low-skill workers in market services (and not home-produced services). Consequently, the rising demand for skill-intensive services and, by extension, market services which accompanies the development process raises the demand and wage premia of high-skill workers. This, in turn, increases the opportunity cost of home production and drives further increases in demand for market services. Unlike here, their paper is more interested in explaining changes in the composition of the US economy rather than productivity growth trends. Duernecker et al. (2017) on the other hand, study the implications of differential productivity growth rates within services and the role of this in driving the post-war productivity growth slowdown in the US. Their key result is that fast-growing and stagnant services are substitutes, meaning that the effects of the Baumol cost disease on productivity growth are unlikely to be as pronounced in the future as in the past. While their paper yields important insights for my question, these are more relevant for advanced country-settings. I take a different approach, focusing primarily on productivity growth in developing countries.

A growing literature exploring services productivity in developing countries has been advocating for employment reallocation between services sub-sectors as a source of productivity growth. Fang & Herrendorf (2021), for example, study the reasons behind China's underdeveloped services sector. They find that the high-skill-intensive services

occupy a much lower share of the economy compared to countries at similar stages of development, concluding that this is driven by output wedges that translate into high output prices. Removing these wedges would allow for the reallocation of services employment to these sub-sectors. Work by [Baccini et al. \(2021\)](#) finds that while services on the aggregate do not have a strong relationship with economic development, this is only true for specific industries within services. They find that high-skill-intensive sub-sectors, for instance, are typically strongly associated with economic development in Africa. While reallocation effects can potentially drive productivity growth in services, one of the empirical trends documented in this paper is that fast-growing countries benefit mostly from ‘within’ sub-sector productivity growth as opposed to reallocation effects (‘between’ channel).

This paper is organised in the following way. In Section 2, I present the three empirical facts on services productivity growth. Section 3 presents a model that sheds light on the mechanisms driving the facts. The calibration and counterfactual exercises follow in Section 4. I conclude with some avenues for further research in Section 5.

## 2 Empirical Facts

This section establishes three stylised facts on service sector productivity growth. First, the variance in services productivity growth is higher among developing countries than advanced countries. Second, services productivity growth is driven mostly by the ‘within’ channel which reflects growth in productivity inside different services activities. The ‘between’ channel – which reflects changes in labour allocation within services – plays a far smaller role. Third, inside the ‘within’ channel, most industries exhibit conditional convergence with the frontier.

### 2.1 Data

The key data source for this section is the well-known GGDC 10-sector database ([Timmer, De Vries & De Vries 2015](#)). The database offers a unique sectoral breakdown of aggregate data on an annual basis spanning back at least 45 years for a total of 42 countries. Data on sectoral-level employment and both nominal and real value-added is available for these countries, and this data broadly encompasses work in both the formal and informal sectors. The sectoral breakdown follows International Standard Industrial Classification (ISIC) Rev. 3.1. As such, the services sector is defined as the aggregation of trade, transport, business, government and personal sub-sectors. In the interest of maximising the number of countries with full time series in the sample, I focus specifically on the years between 1975 and 2009, leaving me with a total of 1,470 country-year observations. For the remainder of this paper, services productivity is defined as:

$$y_t = \frac{RVA_t}{EMP_t} \quad (1)$$

where  $RVA_t$  and  $EMP_t$  are real value added and employment in services at time  $t$ . Since the 10-sector database presents real value added data in local currency units, I divide it by the exchange rate retrieved from the Penn World Tables (PWT) 10.01 ([Feenstra et al. 2015](#)). The level of development of a country is proxied by real GDP per capita which is also calculated using the PWT<sup>1</sup>. Robustness checks for Facts 1 and 2 are performed in Appendices A.1 and A.2 respectively, using the GGDC Economic Transformation Database (ETD) ([Kruse et al. 2022](#))<sup>2</sup>. I also run robustness regressions for Fact 3 in Appendix A.3<sup>3</sup>.

### 2.2 Facts

#### 2.2.1 Fact 1: Variance in Services Productivity Growth is Higher in Developing Countries Than Advanced Ones

The first fact to highlight is that the variance in average services productivity growth between 1975 and 2009 is larger in developing countries compared to their advanced counterparts. Table 1 presents the first and second moments of a cross-section of average services productivity growth between 1975 and 2009 computed by the level of development.

<sup>1</sup>Real GDP per capita is calculated by dividing output-side real GDP at chained PPPs by the population.

<sup>2</sup>The 10-sector database is preferable to the ETD given that it goes further back in time and has a more representative sample of developing countries. The ETD is therefore kept for robustness checks. Relative to the 10-sector database, the ETD contains more developing countries and offers more recent data (up to 2018).

<sup>3</sup>In particular, I run the same regression using the M-estimator.

Average services productivity growth in advanced countries<sup>4</sup> is more than double that of developing ones – though less than 2% which is consistent with the low real GDP growth and large services sectors observed in these countries. Conversely, the variance in average services productivity growth is over four times larger among developing economies than advanced ones.

|                             | mean | variance | N  |
|-----------------------------|------|----------|----|
| <b>Advanced Economies</b>   | 1.35 | 1.06     | 15 |
| <b>Developing Economies</b> | 0.65 | 5.00     | 27 |

Table 1: Mean and Variance of Services Productivity Growth

*Notes.* The mean and variance are computed using the mean year-on-year services labour productivity growth for each country between 1975 to 2009.  $N$  represents the number of advanced and developing countries in the cross-section. The distinction between advanced and developing countries is made following the IMF's classification system.

The same conclusion can be drawn from Figure 1. The countries represented in red are developing countries while the ones in green are advanced countries. It is clear that there is far greater dispersion in services productivity growth among developing countries than advanced ones. A series of countries such as Peru, Senegal, Kenya and Malawi have seen their productivity growth in services, on average, contract between 1975 and 2009 while countries like China and Botswana have seen rapid services productivity growth in services (above 4%). Advanced countries, on the other hand, have a much smaller range. Barring Taiwan, the advanced economies in the cross-section have had their average services productivity growth range between 0 and 3%, justifying the statistics presented in Table 1. These results appear to mostly hold with data from the ETD which I use a sense check<sup>5</sup>.

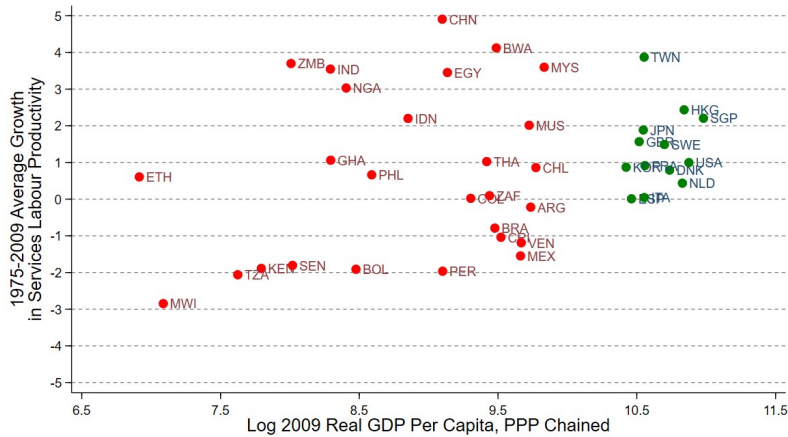


Figure 1: Average Services Productivity Growth by GDP per Capita

*Notes.* The y-axis is mean services labour productivity between 1975 and 2009 while the x-axis is the logarithm of PPP-adjusted real GDP per capita in 2009. Red represents developing countries while green represents advanced economies. The distinction between advanced and developing countries is made following the IMF's classification system.

This fact ultimately serves as the motivation for this paper. Against the backdrop of premature deindustrialisation, factors of production will continue to move from agriculture and manufacturing to services. Figure 2 shows that in virtually all the countries studied, the employment share of services has risen dramatically between 1975 and 2009 regardless of the level of development<sup>6</sup>. Importantly, developing countries are becoming more reliant on services to narrow down cross-country income differences. The heterogeneity observed in average services productivity growth among developing countries suggests that this will not be a straightforward task. The remainder of this paper attempts to understand and measure the factors behind this heterogeneity to better understand how developing countries can avoid an equilibrium of dominant services sector with low (and potentially negative) productivity growth.

<sup>4</sup>The distinction between advanced and developing countries is made following the IMF's classification system.

<sup>5</sup>The results can be found in Appendix A.1.

<sup>6</sup>Unsurprisingly, there appears to be a positive correlation between the level of development and the size of the increase in services employment share across countries. This is in fact consistent with the Kuznets facts which suggest that accompanying the process of economic development is a process of structural change where factors of production move from agriculture to manufacturing and then services.

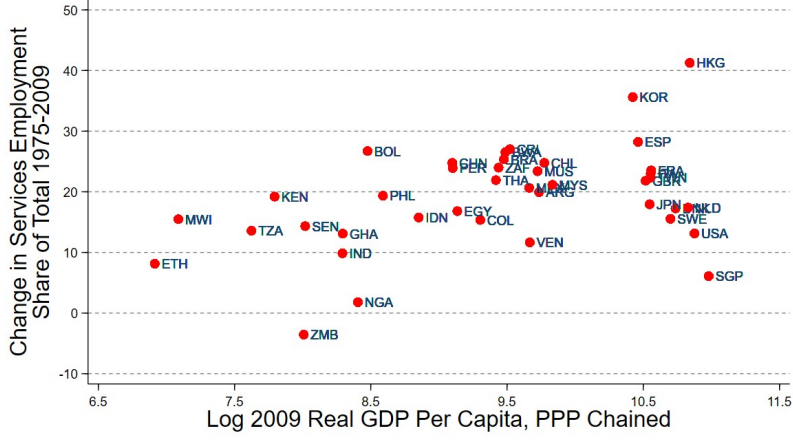


Figure 2: Change in Services Employment Share Between 1975 and 2009

*Notes.* This figure plots the change in the services employment share of total employment between 1975 and 2009 for all countries in the sample against the log of real GDP per capita in 2009 as a proxy for development.

### 2.2.2 Fact 2: Services Productivity Growth is Driven by the ‘Within’ Channel

One would think that countries that were able to reallocate their factors of production (particularly labour) to industries within services that are more productive are those that managed to grow their productivities the fastest in the last few decades. Decomposing the sources of services labour productivity growth would clarify whether this is actually the case. To do so, I follow the decomposition method suggested by [McMillan et al. \(2014\)](#) and, using services sub-sector level data, apply it specifically to the services:

$$\Delta y_t = \sum_{i=n} \theta_{it-k} \Delta y_{it} + \sum_{i=n} y_{it} \Delta \theta_{it} \quad (2)$$

The  $\Delta$  represents the change operator, while  $y_{it}$  is labour productivity in services for services sub-sector  $i$  at time  $t$ .  $\theta_{it}$  is the employment share of services sub-sector  $i$  over the entire services sector’s employment. I set  $t$  to 2009 and  $t - k$  at 1975. Combined, (2) represents a statistical decomposition in services labour productivity into the two main forces that drive sectoral productivity growth – the ‘within’ and ‘between’ forces. The first term in (2) reflects the weighted sum of the ‘within’ force which aggregates the increases in labour productivity inside a given services industry over time. The second term, meanwhile, reflects the ‘between’ factor which captures the change in employment shares or labour reallocation within services over time.

The statistical decomposition above allows one to assess the drivers of services productivity growth across time. Performing this exercise leads to the second stylised fact of this paper, which is that productivity growth in services (or lack thereof) is primarily driven by the ‘within’ as opposed to the ‘between’ component.

Figure 3 plots the share of the increase in service sector labour productivity between 1975 and 2009 driven by the ‘within’ and ‘between’ shares. The takeaway from this is that barring Argentina, all the countries in the cross-section rely on the ‘within’ channel for labour productivity growth as opposed to the ‘between’ channel. Surprisingly, this result appears to be robust to the rate of productivity growth in services, meaning that even countries that saw contractions did so because of declines in the ‘within’ industry productivity. One can even argue that these results suggest that, for many countries that experienced growth in services labour productivity between 1975 and 2009 like Ethiopia, Ghana and some European countries like Sweden and Denmark, their respective growth rates were dragged down by the ‘between’ component. This can be interpreted as the reallocation of labour within services from more productive industries to less productive ones. This is particularly true for China.

Table 2 presents some of the statistics on the decomposition, namely the mean and standard deviations of the share of the services labour productivity increase between 1975 and 2009 driven by the ‘between’ and ‘within’ forces. It confirms that, on average, the ‘within’ component drives almost all of the services labour productivity increase between the two periods. The means of the ‘between’ and ‘within’ channels sum to 0.99, so the remaining 0.01 can be accounted for by an interaction term. The standard deviations, while fairly sizable, suggest that these conclusions

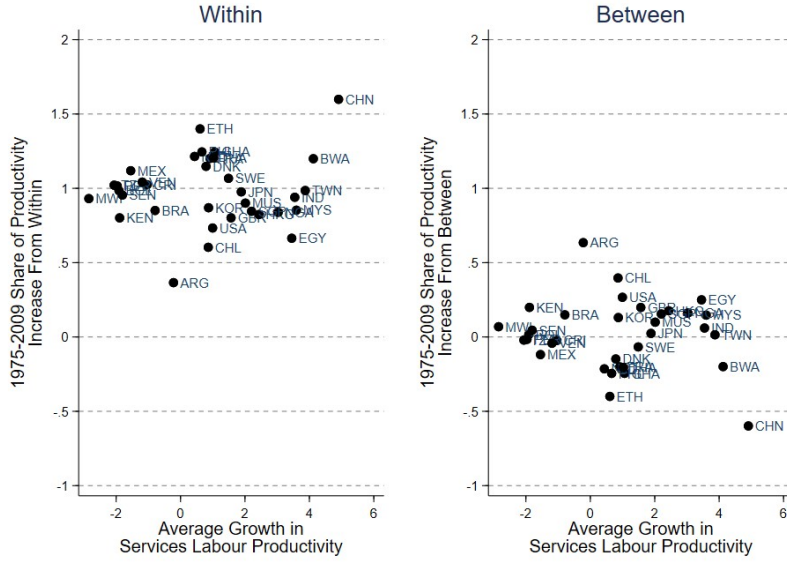


Figure 3: Statistics on the Decomposition of Productivity Increase

*Notes.* This figure plots the results of the statistical decomposition of services labour productivity growth between 1975 and 2009. The y-axis the share of the increase in services labour productivity between 1975 and 2009 driven by the ‘within’ (LHS) and ‘between’ components (RHS). The x-axis is mean year-on-year growth in services labour productivity between 1975 and 2009. Colombia, Zambia, India, Italy, South Africa and Spain are excluded due to excessively large positive and negative shares. The interaction term is not included because it plays a minor role in this exercise.

hold for most of the countries. Running the same decomposition exercise on services productivity data from the ETD as a robustness check yields similar results<sup>7</sup>.

|                | mean | SD   | N  |
|----------------|------|------|----|
| <b>Between</b> | 0.01 | 0.23 | 36 |
| <b>Within</b>  | 0.98 | 0.24 | 36 |

Table 2: Statistics on the Decomposition of Productivity Increase

*Notes.* This table presents the mean and standard deviations of the share of the services labour productivity increase between 1975 and 2009 driven by the ‘between’ and ‘within’ forces. Colombia, Zambia, India, Italy, South Africa and Spain are excluded due to excessively large positive and negative shares.  $N$  represents the number of advanced and developing countries in the cross-section.

These results indicate that to understand the heterogeneity in developing countries’ ability to grow their services productivity, one should examine the sources of productivity growth at the sub-sector level. Reallocation effects, while having the potential to drive productivity growth further, appear to be of second-order importance in explaining the speed of progress in countries like China and Botswana and, at the same time, the poor performance of other countries like Mexico and Malawi.

### 2.2.3 Fact 3: The ‘Within’ Channel’s Effect is Partly Driven by Conditional Convergence

The main takeaways thus far are that heterogeneity in countries’ successes in growing their services labour productivity is far greater among developing countries than their advanced counterparts and that the ‘within’ channel is the dominant driver of productivity growth. What remains is pinning down how exactly the ‘within’ channel operates and drives labour productivity growth in services. Here it is important to distinguish between the two main sources of ‘within’ sector growth which are catch-up and CRS returns. The former would suggest that growth, particularly in developing countries, is underpinned by knowledge diffusion and catch-up of advanced countries’ technologies used in services. The latter, on the other hand, would suggest that labour productivity growth is driven by country-specific trends, namely innovation within sectors that are specific to different countries. To test for the presence of convergence, I follow the semi-elasticity model specification in [Rodrik \(2013\)](#) :

<sup>7</sup>The results can be found in Appendix A.2.

$$\hat{y}_{ict} = -\beta \ln y_{ict-k} + \chi_i + \varepsilon_{ict} \quad (3)$$

where  $\hat{y}_{ict}$  is growth in services labour productivity in services sub-sector  $i$  in country  $c$  between time  $t$  and  $t-k$ . The explanatory variable is  $\ln y_{ict-k}$  which represents the natural logarithm of the initial productivity level at time  $t-k$ . The coefficient of interest is therefore the corresponding  $\beta$ , which is referred to as  $\beta$ -convergence in the cross-country growth literature. Since I pool all services sub-sectors in the dataset, I include sub-sector fixed effects denoted by  $\chi_i$  to absorb industry-specific trends. Finally,  $\varepsilon_{ict}$  is the error term that is specific to each industry, country and year. The essence of this model is to test whether there exists a link between initial labour productivity levels and future growth rates. The presence of convergence would imply a negative and statistically significant coefficient on the the initial productivity level as countries with lower initial productivity should imply higher growth later on. One important note around interpretation is that (3) purposefully excludes country fixed effects because it tests for *unconditional* convergence. In other words, it tests if the gap between two countries' labour productivity would decline (i.e. converge) regardless of these countries' structural characteristics and environment.

Broadly in line with [Rodrik \(2013\)](#), I run three specifications based on (3). The first is the cross-section specification following [Mankiw et al. \(1992\)](#), where initial productivity is the productivity level in year 1975 and the dependent variable is the average growth in productivity level between 1975 and 2005<sup>8</sup>. Given the lack of time dimension, year and industry-year fixed effects are excluded. The second is the panel specification based on [Islam \(1995\)](#). Here I slice the data into three time periods, 1975-1985, 1985-1995 and 1995-2005. The initial productivities are thus of 1975, 1985 and 1995 and the dependent variables are the compound annual growth rates for each decade in between. As such, the number of observations triple and year- and industry-year fixed effects are added to (3). Finally, I run a cross-section specification with just the final decade, where the independent variable is the productivity level in 1995 and the dependent variable is the compound annual growth rate for the subsequent decade.

The results are reported in Table 3. Column (1) presents the results of the cross-section specification, while column (2) presents the panel specification and column (3) presents the cross-section specification with just the final decade. While the coefficient of interest is negative across all three specifications, the results are statistically insignificant. The conclusion of unconditional convergence is categorically rejected. Countries with low initial productivities relative to the frontier (captured by industry fixed effects) do not grow faster than countries with productivities closer to the frontier.

|                  | (1)              | (2)              | (3)              |
|------------------|------------------|------------------|------------------|
| $\ln y$          | -0.31<br>(-1.33) | -0.17<br>(-0.86) | -0.22<br>(-0.72) |
| Industry-Year FE | ×                | ✓                | ×                |
| Industry FE      | ✓                | ✓                | ✓                |
| Year FE          | ×                | ✓                | ×                |
| Country FE       | ×                | ×                | ×                |
| N                | 190              | 570              | 190              |

Table 3: Results of Unconditional Convergence Regressions

*Notes.* This table presents the standard OLS cross-section regression results of (3). The dependent variable is productivity growth in services. Column (1) represents the cross-section specification, column (2) represents the panel specification and column (3) represents the specification keeping the final decade. T-statistics are in parentheses. All standard errors are clustered by country. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

One can also test for conditional convergence by simply adjusting (3) to include country fixed effects:

$$\hat{y}_{ict} = -\beta \ln y_{ict-k} + \chi_i + \gamma_c + \varepsilon_{ict} \quad (4)$$

where  $\gamma_c$  are the country fixed effects. Examining the conditional distribution of the  $\beta$ -convergence coefficient is deemed more appropriate in the cross-country growth empirics literature ([Barro et al. 1991](#)) as it tests for convergence conditioning on countries' characteristics. Including country fixed effects would hold constant variations in countries'

<sup>8</sup>The 30-year time window is aimed at ensuring comparability with [Rodrik \(2013\)](#).

structural characteristics such as institutions and environment. Put differently, conditional convergence implies countries converge to steady states, and the differences in the steady state levels are subtracted away using the country fixed effects.

Table 4 presents the results of the conditional convergence regressions. As before, columns (1), (2) and (3) present the coefficients from the cross-section, panel and cross-section with just the final decade specifications, respectively. The results show a clear change in statistical significance of the coefficients of interest vis-a-vis the coefficients from the unconditional convergence regressions. All specifications are statistically significant at the 1% level, and the coefficient on the panel specification is more than double that of the cross-section specification meaning that it is the most economically significant. Its coefficient implies that a one percentage increase in the 1995 productivity level leads to a 0.28 percentage point decline in the compound annual growth rate in the subsequent decade. Overall, the results in Table 4 indicate the presence of conditional convergence in services.

|                  | (1)                 | (2)                 | (3)                 |
|------------------|---------------------|---------------------|---------------------|
| $\ln y$          | -1.39***<br>(-5.25) | -1.51***<br>(-5.10) | -2.79***<br>(-4.01) |
| Industry-Year FE | ×                   | ✓                   | ×                   |
| Industry FE      | ✓                   | ✓                   | ✓                   |
| Year FE          | ×                   | ✓                   | ×                   |
| Country FE       | ✓                   | ✓                   | ✓                   |
| N                | 190                 | 570                 | 190                 |

Table 4: Results of Conditional Convergence Regressions

*Notes.* This table presents the standard OLS cross-section regression results of equation (4). The dependent variable is productivity growth in services. Column (1) represents the cross-section specification, column (2) represents the panel specification and column (3) represents the specification keeping the final decade. T-statistics are in parentheses. All standard errors are clustered by country. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

These results are confirmed graphically from Figure 4 which plots the conditional convergence regression (panel specification) where the dependent variable is compound annual productivity growth in a given decade and the independent variable is the corresponding initial productivity of that decade. Thus, each observation represents a country-sector-decade. Figure 3 shows a clear negative correlation between the initial productivity level in a given country-sector-decade and subsequent productivity growth. The results are also broadly similar to those found in [Herrendorf et al. \(2022\)](#). The robustness checks confirm these conclusion<sup>9</sup>.

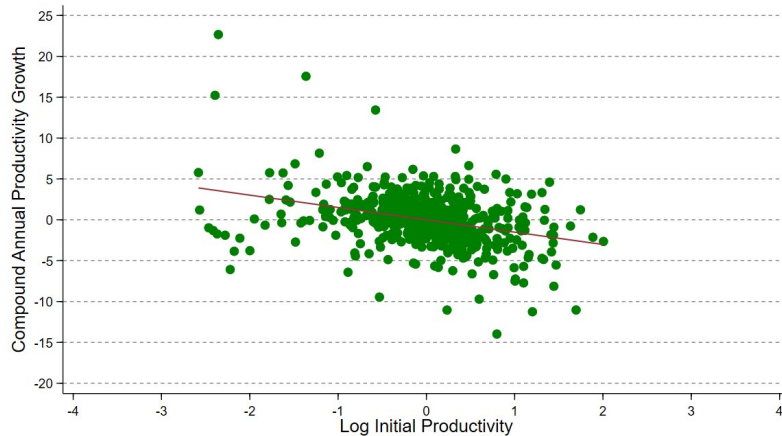


Figure 4: Conditional Convergence in Services

*Notes.* This figure plots the results of the panel specification of the conditional convergence regressions where the dependent variable is the 10-year compound annual growth rate in productivity in a given country-sector-decade. The independent variable is the log of initial productivity level for the country-sector at the start of the decade. Industry, industry-year, year and country fixed effects are included. Each observation is a country-sector-decade. All standard errors are clustered by country.

<sup>9</sup>The results can be found in Appendix A.3.

Reconciling the results in [Rodrik \(2013\)](#) and the general consensus of a lack of evidence of unconditional convergence at the aggregate level in the growth empirics literature with the results presented here is fairly straightforward. Services productivity in developing countries can only converge to the frontier once country-level structural characteristics like institutions and geography are controlled for. Developing countries cannot, therefore, expect to unconditionally converge with the frontier at the aggregate unless the manufacturing sector takes on a bigger share of the economy, though premature deindustrialisation among less advanced economies implies that this is unlikely to be the case going forward. Furthermore, the results here already point to a major difference between the services and manufacturing sectors. The latter can converge with the frontier unconditionally while the former requires these country-specific conditions to be addressed first.

### 3 Model

The purpose of this section is to present a theoretical framework that sheds light on the mechanisms driving the facts presented in the empirical section. The planner economy is populated with one representative household endowed with just one factor of production that is, technology, which is used to produce output in services. The planner faces two problems. The first is an intertemporal problem where the planner must maximise lifetime utility. The second problem involves a static optimisation problem where the planner must maximise productivity growth through the allocation of physical investments. The population is normalised to unity and model variables are expressed in per-worker terms.

#### 3.1 Environment

##### 3.1.1 Utility

The social planner's goal is to maximise the household's lifetime utility. The household's utility function  $u(c) = \log(c)$  is strictly increasing, concave and twice differentiable with  $u'(c) > 0$  and  $u''(c) < 0$ :

$$\max_{\{c_t\}} U = \int_0^\infty e^{-\rho t} \log c_t dt \quad (5)$$

$c_t$  and  $\rho$  represent consumption and the subjective discount factor, respectively. The planner maximises lifetime household utility facing the following resource constraint:

$$y_t = c_t + z_t \quad (6)$$

where  $y_t$  and  $z_t$  represent output in services and physical investment, respectively. This simple maximisation problem already implies an intertemporal trade-off between consumption and investment observed in the standard neoclassical growth model. The planner can increase consumption at the expense of physical investment at time  $t$ , implying slower growth in future output.

##### 3.1.2 Technology

The planner makes two decisions in this model. The first involves the choice of consumption which was just introduced. The second corresponds to the choice of technology used to grow output in services. To see this, first consider the stock of technology which features directly in the production function,  $y_t$ . Technology is used through a one-to-one mapping to produce output in services:

$$y_t = a_t \quad (7)$$

where the key property is monotonicity. In this model, the stock of technology evolves according to the following law of motion:

$$\dot{a}_t = \Gamma(\omega, t, a) z_t - \delta a_t \quad (8)$$

The technology stock depreciates at the rate  $\delta$  which is assumed to be time- and country-invariant and grows using two inputs. The first is  $z_t$  which is the physical investment into technology decided by the planner. The second is  $\Gamma(\omega, t, a)$  which represents the return per unit of investment and depends on the planner's choice between catch-up and constant returns to scale (CRS) technologies:

$$\Gamma(\omega, t, a) = \max_{\{\omega_t\}} [\omega_t \gamma (\bar{a}_t - a_t) + (1 - \omega_t) \eta] \quad (9)$$

and:

$$a_0 > 0 \quad (10)$$

The social planner chooses  $\omega_t$  which is simply a distribution parameter. It determines the shares of the physical investment allocated to the catch-up and CRS technologies. The choice is endogenous in that the planner makes the decision simply by comparing the marginal gains between both forms of technology growth. The first term in the square bracket in (9) represents the marginal gains from the catch-up technology which is determined by how far the current technology stock is from the frontier technology stock,  $\bar{a}_t$ .  $\gamma$  is a growth efficiency parameter which captures the institutional and geographical environment and is therefore time-invariant<sup>10</sup>. This could be interpreted to capture the country-level fixed effects in the previous section. The alternative decision is the CRS technology where technology grows at a constant rate  $\eta$  which does not vary across countries. Finally, the frontier is assumed to grow at a constant rate  $g_{\bar{a}}$ :

$$\bar{a}_t = \bar{a}_0 e^{g_{\bar{a}} t} \quad (11)$$

It is important to reiterate that  $\Gamma(\omega, t, a)$  captures the channel of productivity growth at the sectoral level that abstracts from the factor reallocation effects and focusing exclusively on improvements in the production technology ('within' channel). One can already see that this incorporates the final fact which documents that some countries are able to catch-up with their services sector while others do not. This heterogeneity is dealt with using the growth efficiency parameter,  $\gamma$ , which varies by country. The presence of  $\gamma$  also naturally replicates the first fact which documents the dispersion in average growth rates in services productivity among developing countries. Another thing to note is that  $\Gamma(\omega, t, a)$  shrinks as  $a_t$  rises because of decreasing marginal returns from the catch-up technology. Once the planner transitions to the CRS technology, the return per-unit of investment is simply just a function of the CRS technology growth rate  $\eta$ . The social planner therefore faces a static optimisation problem of maximising technology growth and then, conditional on this choice, maximises the household's welfare. Note that the no-ponzi condition is also imposed for the intertemporal problem:

$$\lim_{t \rightarrow \infty} \left[ a_t e^{-\int_0^t r_s ds} \right] \geq 0 \quad (12)$$

The standard interpretation holds, which is that the no-ponzi condition ensures that in the limit, the net present value discounted at rate  $r(s)$  of the household's stock of technology,  $a_t$ , cannot be negative. This constraint rules out the consumption path that involves accumulating infinite debts.

### 3.2 Equilibrium

Given the model environment, the following definition of a competitive equilibrium is imposed:

**Definition 3.1 (Equilibrium)** *A planner equilibrium consists of paths of consumption, technology and technology choice  $[c_t, a_t, \omega_t]_{t=0}^{\infty}$  such that:*

1. *The social planner maximises the net present value of the household's utility by choosing optimal paths of investment and consumption  $[z_t, c_t]_{t=0}^{\infty}$ ;*

---

<sup>10</sup>I impose a restriction that  $\gamma$  is equal to or greater than 0.25. This will prove useful for defining the convergence criteria in the next sub-section.

2. The planner maximises technology growth by choosing the path of allocation to catch-up technology  $[\omega_t]_{t=0}^{\infty}$  taking the frontier technology, the current level of technology and the two investment returns as given;
3. There is market clearing such that the budget constraint binds.

The social planner makes two decisions, beginning with the choice of  $\omega_t$  for the optimal allocation of the physical investment. The solution to the static problem is a sequence of the following actions:

$$\omega_t^* = \begin{cases} 1 & \text{if } \gamma[\bar{a}_t - a_t] > \eta \\ \text{mix} & \text{if } \gamma[\bar{a}_t - a_t] = \eta \\ 0 & \text{if } \gamma[\bar{a}_t - a_t] < \eta \end{cases} \quad (13)$$

Since  $a_t$ ,  $\eta$  and  $\bar{a}_t$  are all given, the planner simply compares the gains from the catch-up and CRS technologies. The social planner chooses  $\omega_t = 1$  if the current technology stock is far below the frontier to the extent that it outweighs the potential returns from the CRS technology,  $\eta$ <sup>11</sup>. Conversely, the planner can choose  $\omega_t = 0$  if it is too close to the frontier that it has less to gain than if the planner allocates the physical investment to the CRS technology. The only exception to this is if the gains from the catch-up and CRS technologies are exactly equal. While this is a rare scenario, one can think about this as a country whose productivity in services is high, but not quite at the frontier. It can potentially catch-up, but it can also employ the CRS technology with equal growth prospects. In this case, the planner would opt for a mixed strategy. Any share of  $\omega_t$  would maximise technology growth so the planner can randomise.

### 3.2.1 Transitional Dynamics

The transitional dynamics in this model refer to the case where the planner's optimal allocation of the physical investment is to the catch-up technology. The equilibrium consumption and investment functions will naturally differ in this case compared to the case where the technology stock has reached the frontier and the planner prefers to use the CRS technology ( $\omega^* = 0$ ), in which case the economy enters a balanced growth path and a steady state can be computed using normalised values for consumption and physical investment.

**Proposition 1** *The equilibrium paths of consumption and technology stock  $[c_t, a_t]_{t=0}^{\infty}$  for the case where the planner prefers the catch-up technology ( $\omega^* = 1$ ) evolve according to the following equations:*

1. 
$$\frac{\dot{c}_t}{c_t} = -\rho - \frac{(\bar{a}_t g_{\bar{a}} - \gamma(\bar{a}_t - a_t)z_t + \delta a_t)}{(\bar{a}_t - a_t)} - \frac{\dot{\mu}_t}{\mu_t} \quad (14)$$

2. 
$$\dot{a}_t = (a_t - c_t)\Gamma(\omega, a, t) - \delta a_t = z_t(\bar{a}_t - a_t)\gamma - \delta a_t \quad (15)$$

where  $\frac{-\dot{\mu}_t}{\mu_t} = \gamma(\bar{a}_t - a_t) - (a_t - c_t)\gamma - \delta$

**Proof 1** See Appendix B.1 ■

(14) is the standard Euler equation which dictates the equilibrium growth in consumption in the economy.  $\mu_t$ , the co-state variable in this optimal control problem, represents the shadow value of accumulating one more unit of wealth. The discount rate, meanwhile, is subtracted exactly as in the neoclassical growth model. A more impatient household would mean less consumption growth. Next, (15) reflects the optimal choice of the static problem where the planner prefers the catch-up technology. As such, growth in productivity will be a function of the physical investment and distance to the frontier. The further away the domestic productivity is from the frontier, the faster productivity growth is.

### 3.2.2 Balanced Growth Path

Thus far it has been assumed that  $\gamma$  is large enough such that the technology stock eventually catches up to the frontier. Provided that this is the case, the planner eventually switches from the catch-up technology to the CRS technology ( $\omega^* = 0$ ). The growth dynamics thereafter are important because they define the limiting case.

<sup>11</sup> Assuming  $\gamma$  is greater than 1 then implies that “catch-up” is defined as a distance to the frontier at least as small as  $\eta$ , if not smaller.

**Proposition 2** *The equilibrium paths of consumption and technology stock  $[c_t, a_t]_{t=0}^{\infty}$  for the case where the planner prefers the CRS technology ( $\omega^* = 0$ ) evolve according to the following equations:*

1.

$$\frac{\dot{c}_t}{c_t} = \eta - \delta - \rho \quad (16)$$

2.

$$\dot{a}_t = (a_t - c_t)\Gamma(\omega, a, t) - \delta a_t = z_t \eta - \delta a_t \quad (17)$$

**Proof 2** *See Appendix B.2 ■*

Proposition 2 presents the equilibrium paths of consumption and technology once the planner decides to employ the CRS technology. One important observation to note is that consumption growth is increasing in  $\eta$ , the constant rate at which physical investment is turned into productivity growth. The same can be noted about technology growth. To ensure that the planner switches just once, a new simplifying assumption will be introduced for the rest of the paper.

**Assumption 1** *The growth rate of the frontier technology is equal to the growth rate of technology from the CRS technology:*

$$g_{\bar{a}} = \frac{\dot{a}_t}{a_t} = \frac{\dot{\bar{a}}_t}{\bar{a}_t} \quad (18)$$

A key implication of Assumption 1 is that it suggests that both the frontier and the planner's economy share the same  $\eta$  and  $\delta$  provided that the frontier economy's technology stock evolves through the same law of motion as the domestic economy. Indeed, Assumption 1 only formalises the model environment's initial premise which stated that both  $\eta$  and  $\delta$  are country-invariant. To see this, the law of motion for technology stock can be substituted into (18) to yield:

$$g_{\bar{a}} = \frac{\dot{a}_t}{a_t} = \frac{\dot{\bar{a}}_t}{\bar{a}_t} = \frac{\eta z_t - \delta a_t}{a_t} \quad (19)$$

The domestic economy's technology stock thus first grows through the catch-up technology ( $\omega^* = 1$ ) provided its growth efficiency,  $\gamma$  is large enough. Once the domestic economy's technology reaches the frontier, the planner switches to the CRS technology ( $\omega^* = 0$ ) and both the frontier and domestic economy grow at the same rate according to (19).

### 3.2.3 Steady State

Having established that both the frontier and domestic economy grow at the same rate under the balanced growth path, the remaining task is to solve for steady state levels of consumption and physical investment. I assume that this model is consistent with the Kaldor facts, so in the presence of the balanced growth both consumption and output must grow at the same rate in order for the consumption-output ratio to be constant. By virtue of the resource constraint (6), physical investment must also grow at the same rate. Since physical investment, consumption and output levels grow at the same rate, one can examine the steady state by normalising consumption and investment by output that is, examine consumption, investment and output in efficiency units.

Recognising that both consumption and output grow at the same rate, one can equalise the consumption Euler equation with the frontier growth rate of output to solve for the steady state physical investment level which is simply physical investment normalised by output:

$$g_{\bar{a}} = \frac{\dot{c}_t}{c_t} = \eta - \delta - \rho = \frac{\eta z_t - \delta a_t}{a_t} \quad (20)$$

With some basic algebra involving (20) and the resource constraint (6), steady-state output is normalised to 1 and steady-state consumption and physical investment levels can be represented in closed form as:

$$\frac{z_t}{a_t} = \frac{\eta - \rho}{\eta} \quad (21)$$

$$\frac{c_t}{a_t} = \frac{\rho}{\eta} \quad (22)$$

Given that consumption, physical investment and output all grow at the same rate, consumption and physical investment in efficiency units, equations (21) and (22) respectively, are constant over time provided that the domestic economy has reached the frontier and is on the balanced growth path.

## 4 Quantitative Analysis

This section uses the model to interpret the data through a calibration exercise. The results are then used to run a services productivity growth variance decomposition as well as two counterfactual exercises. The first quantitatively assesses the importance of the three different sources of productivity growth (namely growth efficiency, distance to the frontier and physical investment) in a given time period. The second explores the effect of changes in the growth efficiency parameter on the trajectory of productivity growth over time.

### 4.1 Data

The data used for calibration and by extension the counterfactual exercises is mainly retrieved from the 2016 release of the GGDC WIOD ([Timmer, Dietzenbacher, Los, Stehrer & De Vries 2015](#)). The WIOD is a matrix that contains data on linkages between 56 sectors (ISIC Rev. 4) for 43 countries. Output from a sector can be used either as input for another country-sector or as final expenditure for a country. The WIOD maps these linkages for different country-sectors, allowing one to focus exclusively on the service economy. The benefit of this database vis-a-vis the GGDC 10-sector database is that on top of providing data on services value-added, the matrix allows me to extract the consumption and investment shares of services value added (output net of intermediate inputs) which is required to calibrate the growth efficiency parameter. I make use of the price deflators in the PWT 10.01 ([Feenstra et al. 2015](#)) to turn nominal USD data into real and PPP-adjusted data<sup>12</sup>. Per-worker calculations rely on services employment figures from the GGDC 10-sector database ([Timmer, De Vries & De Vries 2015](#)).

Of the 43 countries in the WIOD, I keep the 14 that overlap with the initial GGDC 10-sector database: Brazil, China, Denmark, Spain, France, Great Britain, Indonesia, India, Japan, Mexico, New Zealand, Sweden, Taiwan and the USA (only as a frontier benchmark). These countries ensures a good balance of developed and advanced economies in the mix. The performance of this subset of countries in terms of services productivity growth varies greatly. Brazil and Mexico saw, on average, contractions in average services productivity between 1975 and 2009 while China was, on average, the fastest growing of the 42 countries I initially considered in the 10-sector database. Furthermore, I focus on the years 2000 to 2005 – the five years where the WIOD and the time range considered in Section 2 overlap.

### 4.2 Calibration

The calibration strategy relies first and foremost on obtaining services value added. I focus on value-added as a measure of output since the theoretical framework abstracts from any intermediate inputs. For consistency, consumption that is taken as input in the utility function (5) must then refer to the value-added component of final expenditure in services as opposed to just final expenditure in services which as in [Herrendorf et al. \(2013\)](#) I will refer to as consumption value-added in services<sup>13</sup>. To obtain the consumption value-added in services, I implement the method in [Herrendorf et al. \(2013\)](#)<sup>14</sup>. This exercise yields a time series of services consumption and investment value-added in services for 14 countries between 2000 and 2005 which is used to calibrate the model parameters of interest.

The parameter values for the first five parameters are presented in Table 5. I begin by assigning the discount factor a value of 0.020, in line with most of the macro literature. I then use (22) to compute  $\eta$  – the per-unit investment

<sup>12</sup>I divide nominal USD value added in services for a given country by the ratio of the price level of consumption in USD terms of the given country to the 2017 price level of output in the US. Assuming that the price levels of consumption and services are similar in a given country - an assumption motivated by the fact that services are mostly consumed - this prices quantities in US prices constant at base year 2017.

<sup>13</sup>The key distinction between final expenditure and consumption value added can be illustrated through the example in [Herrendorf et al. \(2013\)](#). Buying a shirt in the final expenditure component is considered simply consumption of a manufactured good. The consumption value-added approach instead breaks this down into consumption of agriculture (through the cotton input), manufacturing (through the process of manufacturing the shirt) and services (through the distribution of the shirt from the factory to the store). [Herrendorf et al. \(2013\)](#) show that both the final expenditure and consumption value-added approaches are linked through the input-output tables.

<sup>14</sup>The exact details of this method can be found in their appendix, B.2.

return of the CRS technology. Note that the steady state analysis corresponds to a country that is at the frontier and is therefore on a balanced growth path. The US tops the group in terms of having the highest PPP-adjusted real services value added per worker for each year between 2000 and 2005 so I consider it to be the frontier. The US's share of services value added being (invested) consumed, known henceforth as the (investment-) consumption-output ratio, is fairly stable, ranging between (16-17%) 83-84% between 2000 and 2005 with an average of (16.5%) 83.5%. Calibrating  $\eta$  to match the steady state condition (22) for the US yields a value of 0.024. The US PPP-adjusted real services value added per worker grew by a compound annual rate of 2.2% between 2000 and 2005, informing the frontier growth rate value  $g_{\bar{a}}$ . The depreciation rate,  $\delta$ , is set to 1%. The initial frontier technology stock, meanwhile, is assigned a value of 1.000 since PPP-adjusted real services value added per worker is normalised by the frontier.

| Parameter     | Description                       | Value       |
|---------------|-----------------------------------|-------------|
| $\rho$        | discount factor                   | 0.020       |
| $\eta$        | CRS technology return             | 0.024       |
| $g_{\bar{a}}$ | frontier growth rate              | 0.022       |
| $\delta$      | depreciation rate                 | 0.010       |
| $\bar{a}_0$   | initial frontier technology stock | 1.000       |
| $a_0$         | initial domestic technology stock | see table 6 |
| $\gamma$      | growth efficiency                 | see table 6 |

Table 5: Parameter Values  
*Notes.* This table presents the parameter values for the discount factor, the CRS technology return, the frontier growth rate and the initial frontier technology stock. See text for more details.

The growth efficiency parameter is country-specific, so these are presented in Table 6 along with the initial output, initial investment rate, initial investment level and 2000-2005 compound annual growth rates of output for each country. Output for each country is retrieved using PPP-adjusted real services value added per worker normalised to that of the US – the frontier. Its initial value would therefore correspond to the first point of available data which is the year 2000 though I take a 2000-2001 average in the interest of reducing noise, noting that this is too short of a time span for one to observe meaningful changes that can be attributed to the catch-up effect. The first column in Table 6 is therefore the 2000-2001 average PPP-adjusted real services value added per worker divided by the value of its US counterpart. The second column is the average 2000-2001 services investment-value added ratio while the third column, initial physical investment, is the product of the first and second columns. The fourth column is the 2000-2005 compound annual growth rate in PPP-adjusted real services value added per worker. I calibrate the growth efficiency parameter ( $\gamma$ ) to match the compound annual growth rate in PPP-adjusted real services value added per worker between 2000 to 2005 using (15) and these country-specific values. The results are presented in the fifth column.

|            | Initial<br>Output | Initial<br>Investment Rate | Initial<br>Investment | Output<br>Growth Rate | Growth Efficiency<br>Parameter |
|------------|-------------------|----------------------------|-----------------------|-----------------------|--------------------------------|
| <b>IND</b> | 0.106             | 0.032                      | 0.003                 | 0.037                 | 1.669                          |
| <b>CHN</b> | 0.060             | 0.141                      | 0.008                 | 0.092                 | 0.771                          |
| <b>JPN</b> | 0.632             | 0.110                      | 0.070                 | 0.017                 | 0.658                          |
| <b>MEX</b> | 0.391             | 0.082                      | 0.032                 | 0.021                 | 0.623                          |
| <b>GBR</b> | 0.586             | 0.135                      | 0.079                 | 0.023                 | 0.598                          |
| <b>FRA</b> | 0.696             | 0.162                      | 0.113                 | 0.016                 | 0.522                          |
| <b>TWN</b> | 0.777             | 0.127                      | 0.099                 | 0.005                 | 0.513                          |
| <b>IDN</b> | 0.100             | 0.108                      | 0.011                 | 0.039                 | 0.507                          |
| <b>ESP</b> | 0.596             | 0.102                      | 0.061                 | 0.006                 | 0.397                          |
| <b>DNK</b> | 0.435             | 0.141                      | 0.061                 | 0.006                 | 0.250                          |
| <b>SWE</b> | 0.445             | 0.141                      | 0.063                 | 0.001                 | 0.250                          |
| <b>NLD</b> | 0.542             | 0.238                      | 0.129                 | 0.003                 | 0.250                          |
| <b>BRA</b> | 0.160             | 0.072                      | 0.011                 | -0.007                | 0.250                          |

Table 6: Country-Specific Calibration Results for Growth Efficiency  
*Notes.* This table presents the results of the calibration exercise. The growth efficiency parameter ( $\gamma$ ) is calibrated to match the growth rate in the 2000-2005 compound annual PPP-adjusted real services value added per worker. 2000-2001 average output and investment are used as inputs and are retrieved from the GGDC WIOD. See text for more details on calibration strategy.

Table 6 reveals variation in the initial output level among the countries considered. Advanced countries such as

Spain and Japan have PPP-adjusted services value added per worker equivalent to around three-fifths of that in the US, while in developing countries like India and Indonesia, this value is about a tenth. Another interesting feature about Table 6 is that the countries under consideration have varying degrees of success in growing their services value added per worker. This is particularly problematic for countries further away from the frontier, where premature deindustrialisation is threatening to widen cross-country income differences. In Brazil, for instance, services value added per worker contracted by a compound annual rate of 0.7% between 2000 and 2005 while China grew by 9.2%. This is broadly consistent with the empirical facts, as Brazil was indeed one of the countries showing an average contraction while China was the fastest growing among the countries considered (as is the case here). An important question arises from these observations: What is behind the growth differences among developing countries? Indeed, this question goes back to the first empirical fact, where it was established that there is greater variance in developing countries' ability to grow their services productivity growth vis-a-vis advanced economies.

To make sense of the calibration results for growth efficiency, it is useful to recall that according to (15), output growth is monotonically increasing in three country-specific factors: the distance to the frontier ( $\bar{a}_t - a_t$ ), the time-invariant growth efficiency parameter ( $\gamma$ ) and the investment ratio ( $z/a$ ) (which determines  $z_t$ ). The country with the highest growth efficiency value is India. While initially surprising, this reflects the fact that the country managed to grow at a compound annual growth rate of 3.7% despite having the lowest investment rate of all the countries considered. Indeed, India's growth efficiency parameter is larger than that of China's despite the latter's ability to grow almost three times as fast. Unlike India however, China has a much higher investment rate and benefits from a stronger catch-up effect thanks to a greater distance from the frontier inducing a comparatively lower growth efficiency value. Several advanced economies feature in the middle of the pack. Japan, for instance, has a growth efficiency level of 0.658 while for Great Britain and France this value is 0.598 and 0.522, respectively. Indeed, their relatively lower growth efficiency levels compared to India and China reflects the fact that the speed of their output growth did not match their relatively high investment rate. Brazil is among the countries with the lowest growth efficiency levels, in large part due to its observed contraction in output.

The distribution of growth efficiency levels is a key result in this paper. Growth efficiency is by construction time-invariant and is meant to capture persistent structural characteristics that vary between countries. It is motivated by the empirical results of heterogeneous experiences in services productivity growth in developing countries (Fact 1) and conditional convergence within services across countries (Fact 3). This could represent differences in institutional factors that create a growth-enhancing environment for services or geographical features of a country that support services productivity growth. Before proceeding, it is important to highlight a key implication from Table 6 which will be further explored in the counterfactual exercises. The fact that India and China maintain higher growth efficiency levels compared to advanced economies like Japan and France indicates that the advanced economies have outpaced *some* poorer countries not because of these structural characteristics but through larger investment rates. With an equal distance to the frontier and investment rate, these results indicate that India and China should catch-up to the frontier faster than most advanced economies. In contrast, compared to other countries like Brazil, Great Britain and France hold more favourable time-invariant characteristics that have induced them to grow their services productivity faster. The second counterfactual exercise in particular will offer quantitative evidence in favour of this argument.

## 4.3 Quantitative Exercises

### 4.3.1 Variance Decomposition

The model and calibration results can be used to rationalise the first empirical fact in Section 2 which documents the larger variance in average productivity growth in services among developing economies vis-à-vis advanced ones. Through the lens of the model, a variance decomposition can be performed using the sample of developing countries considered in the calibration results, allowing one to quantify the most important drivers of the dispersion in productivity growth. Noting that all developing countries prefer the catch-up technology, the law of motion for domestic technology and by extension output will evolve according to (15). Setting the time- and country-invariant depreciation rate to 0, dividing both sides by  $a_t$ , taking logs and applying the variance operator yields a linear decomposition of the variance in log productivity growth in services into the variance of each growth driver as well as the corresponding covariance terms.

The results of the variance decomposition for developing countries' productivity growth between 2000 and 2005 are presented in Table 7. In terms of the individual growth drivers, much of the variance in productivity growth is driven

| Variances                               |       | Covariances                 |       |
|-----------------------------------------|-------|-----------------------------|-------|
| Investment ( $z_t$ )                    | 17.7% | $(\gamma, z_t)$             | 17.9% |
| Growth Efficiency ( $\gamma$ )          | 6.1%  | $(\gamma, \bar{a}_t - a_t)$ | 2.1%  |
| Dist. to Frontier ( $\bar{a}_t - a_t$ ) | 0.8%  | $(z_t, \bar{a}_t - a_t)$    | 6.2%  |
| Initial Output ( $a_t$ )                | 13.2% | $(\gamma, a_t)$             | 7.5%  |
|                                         |       | $(\bar{a}_t - a_t, a_t)$    | 6.5%  |
|                                         |       | $(z_t, a_t)$                | 22.1% |

Table 7: Results of Variance Decomposition

*Notes.* This table presents the results of the variance decomposition. The percentages in the table reflect the share of the variance in productivity growth is driven by the corresponding variance/covariance term. Brazil is excluded from this analysis because of it recorded an average contraction.

by both investment (17.7%) and the growth efficiency level (6.1%). The distance to the frontier appears to play a miniature role in explaining the variation in growth, accounting for less than 1%. This is intuitive, given that most of the services productivity levels of developing countries in the sample are almost equally far away from the frontier. In terms of the covariance terms, the most significant ones include the covariance between the growth efficiency level and the investment level (17.9%) as well as the covariance between investment and output (22.1%). Combined, these account for two-fifths of the total variation in productivity growth among developing countries.

#### 4.3.2 Counterfactual Exercise 1: How Can Countries Increase Services Productivity Growth?

My theoretical framework establishes three sources of productivity growth in services: growth efficiency, distance to the frontier (which induces a ‘catch-up’ effect) and the investment rate. The purpose of this counterfactual exercise is to quantitatively assess the effect of changes to each growth driver on the average productivity growth in services of the five developing countries in my sample: Brazil, China, India, Indonesia and Mexico. In particular, I assign the maximum values of each growth driver one at a time for each of the five countries with the hope of shedding light on the most important sources of productivity growth in services. Both the maximum and minimum values of the growth drivers observed among the developing countries under consideration are presented in Table 8.

| Symbol              | Interpretation       | Maximum | Minimum |
|---------------------|----------------------|---------|---------|
| $\gamma$            | growth efficiency    | 1.669   | 0.250   |
| $(\bar{a}_t - a_t)$ | distance to frontier | 0.940   | 0.609   |
| $z_t/a_t$           | investment rate      | 0.141   | 0.032   |

Table 8: Maximum and Minimum Values of Services Productivity Growth Drivers

*Notes.* This table presents the maximum and minimum values for the three growth drivers among the developing countries under consideration. The maximum values are used for the first counterfactual exercise.

The results of this counterfactual exercise are presented in Table 9. All the numbers in the table represent the 2000-2005 compound annual growth rates in PPP-adjusted services value added per worker. The first column is the observed growth rate in the GGDC WIOD data. The second column assigns the maximum growth efficiency observed in a developing country (India with a value of 1.669) to each country, keeping constant the observed distance to the frontier and investment rate of each country. The third column assigns all countries the maximum distance to the frontier observed in a developing country (China with a value of 0.940) and growth efficiency, keeping constant the observed investment rate of each country. Finally, the last column assigns the maximum distance to the frontier, growth efficiency and investment rate (China with a value of 0.141) and, as such, all developing countries have matching growth rates. I interpret this to be the maximum potential growth rate that would be observed if each developing country was to get the highest value for each growth driver observed in the data for developing countries.

It is easy to see that changes to the growth efficiency and investment rates yield the biggest differences in growth rates. Changing the growth efficiency level of all countries to the maximum observed value (Column 2) brings average growth up from 3.6% to 11.3%. Individually, China appears to benefit the most with productivity growth rising by over 10 percentage points. This is unsurprising, given that China led the pack in terms of distance to the frontier and the investment rate. Brazil, meanwhile, sees the growth rate turn from -0.7% to 9.1%. India’s expansion is unchanged because it had the (maximum) growth efficiency level that all countries were assigned. Assigning the counterfactual investment rate observed in a developing country (Column 3) appears to have a similar effect. This induces mean average growth of 18.7% compared to 11.3%. The biggest change happens in India because it had the

|             | Observed      | Counterfactual: Maximum Values |                              |                                                  |
|-------------|---------------|--------------------------------|------------------------------|--------------------------------------------------|
|             |               | $\gamma^{max}$                 | $\gamma^{max} + (z/a)^{max}$ | $\gamma^{max} + (z/a)^{max} + (\bar{a}_t - a_t)$ |
| <b>IDN</b>  | 0.037         | 0.037                          | 0.201                        | 0.210                                            |
| <b>CHN</b>  | 0.092         | 0.212                          | 0.210                        | 0.210                                            |
| <b>MEX</b>  | 0.021         | 0.073                          | 0.134                        | 0.210                                            |
| <b>IDN</b>  | 0.039         | 0.151                          | 0.202                        | 0.210                                            |
| <b>BRA</b>  | -0.007        | 0.091                          | 0.188                        | 0.210                                            |
| <b>Mean</b> | 0.036<br>(0%) | 0.113<br>(44%)                 | 0.187<br>(86%)               | 0.210<br>(100%)                                  |

Table 9: Counterfactual Growth Rates for Developing Countries

*Notes.* This table presents the observed and counterfactual growth rates of the 2000-2005 compound annual PPP-adjusted real services value added per capita. The percentages reflect the distance covered between the observed average growth rate and the maximum potential growth rate.

lowest investment rate among the five countries – the growth rate almost triples. Both Brazil and Mexico’s growth rates double, suggesting major gains from raising investment. Finally, assigning all countries the maximum distance to the frontier in addition to the maximum growth efficiency level and investment rate (Column 4) yields an average growth rate of 21.0% which can be read as the maximum potential growth rate for a developing country.

In terms of closing the distance between the average observed growth rate and the maximum potential growth rate, over four-fifths of this would be covered if countries reached the maximum growth efficiency level and investment rate. This is observed from the fact that average growth rises from the observed value of 3.6% to a counterfactual average of 18.7%. Put differently, eliminating variation in both the investment rate and growth efficiency level is critical if developing countries as a whole were to avoid a scenario of a large and slow-growing services sector that threatens to widen cross-country income differences.

#### 4.3.3 Counterfactual Exercise 2: How Does Growth Efficiency Affect the Trajectory of Services Productivity Level?

The previous counterfactual highlights the importance of the growth efficiency level and investment rate for accelerating developing countries’ productivity growth in services. Crucially though, the investment rate which determines the level of investment,  $z_t$ , would follow the optimal sequence that is dictated by the Euler equation (14). This should be interpreted as the optimal sequence of consumption and, by virtue of the resource constraint, investment. Over- and under-investment relative to this would be interpreted as being sub-optimal according to the model solutions.

In the second counterfactual, I therefore focus exclusively on changes to the growth efficiency level on the trajectory of services productivity growth. The trajectory of the frontier matches the data, and the associated parameters are assigned values from Table 5. Each country is assigned the same initial output and investment rate, but the growth efficiency levels vary from country to country in line with the calibration results in Table 6. Assigning various growth efficiency values holding constant the initial output and investment levels enables one to isolate the impact of growth efficiency on the trajectory of services productivity. Thus, unlike the first counterfactual exercise which focuses on the cross-sectional distribution of services productivity growth, this exercise is more concerned with the trajectory of services productivity over time.

|            | $\gamma$ | $\tau$ |
|------------|----------|--------|
| <b>IND</b> | 1.669    | 8      |
| <b>CHN</b> | 0.771    | 13     |
| <b>FRA</b> | 0.522    | 13     |
| <b>ESP</b> | 0.397    | 14     |
| <b>BRA</b> | 0.250    | 16     |

Table 10: Counterfactual Times to Convergence by Growth Efficiency

*Notes.* This table presents the counterfactual times to convergence.  $\tau$  is the number of time periods required for convergence. Each row was assigned the same investment rate and distance to the frontier. The growth efficiency value is the only parameter that changes.

Table 10 presents the results of the second counterfactual exercise. The first column is the growth efficiency value that

corresponds to each country, where a mix of advanced and developing economies are considered. The second column reveals the number of time periods required for convergence, which is represented by  $\tau$ . Convergence here is dictated by (13). In other words,  $\tau$  is the number of periods that the social planner actively prefers the catch-up technology over the CRS. It is clear that the number of time periods required for convergence falls with the growth efficiency level. Brazil has the lowest possible growth efficiency level (0.250) and therefore takes 16 time periods to converge, while India converges within just 8 time periods given its growth efficiency level of 1.669. These results are reinforced graphically through Figure 3. The solid blue line represents the trajectory of the normalised USD PPP-adjusted services value added per worker of the US which I consider to be the frontier in my calibration exercise. The dashed lines represents the transitional dynamics using the growth efficiency levels of each country under consideration. A higher growth efficiency level accelerates the rate of convergence.

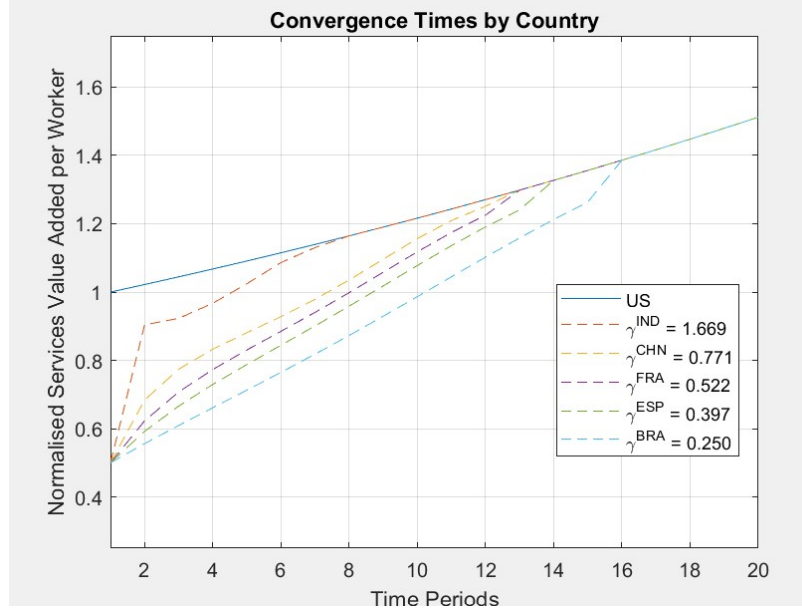


Figure 5: Plot of Output Sequences by Country

*Notes.* The x-axis represents the number of time periods. The y-axis is the PPP-adjusted USD services value added per worker of each country normalised to that of the US in the first time period. The solid line represents the trajectory of the frontier (US). Barring the US, all countries are given the same starting point (output and investment rate) and vary only by growth efficiency level.

These results have important implications for explaining the distribution of services productivity levels across countries. India and China have higher growth efficiency levels than all advanced countries considered in my sample. Accordingly, these countries should converge faster than advanced economies provided that all countries face the same initial productivity level and investment rate. This is, however, inconsistent with what is observed in the data; France and Great Britain have productivity levels far closer to the frontier than India and China. This is confirmed in Table 6. A candidate explanation for this observation is that the advanced economies had consistently higher investment rates than China and India that enabled them to outpace these countries' convergence rates. For Brazil, on the other hand, it is natural that Great Britain and France reach the frontier faster. Brazil's growth efficiency level is half that of France and Great Britain that it would have to compensate with extremely high investment rates. Brazil's observed investment rate in services (also presented in Table 6) is nevertheless also half that of France and Great Britain's which justifies its comparatively lower productivity level and slower convergence rate.

Why then do cross-country gaps in service productivity level still exist if Figure 3 implies convergence in at most 16 time periods (years)? This counterfactual exercise has abstracted from optimal policy. In other words, while consumption and investment are updated according to (14), the initial investment rate is not necessarily the optimal starting point that would bring the economy on the unique saddle-path stable trajectory. It is not inconceivable that the starting investment rate is too high and, in turn, artificially reduces  $\tau$ . Developing countries that have, on average, been seeing contractions in services productivity growth according to the first empirical fact may in fact be investing too little in their services sector compared to the optimal. Indeed, this counterfactual exercise solely attempts to illustrate the importance of the growth efficiency level in determining the trajectory of services productivity levels, and it appears that the gains from increases in growth efficiency can be substantial.

## 5 Conclusion

Can services replace manufacturing as a growth escalator for developing countries? The short answer is no – at least not in the way manufacturing used to. In other words, the unconditional convergence result does not hold for services. Does this mean that the aggregate cross-country income gap is set to persist in the long-run? The answer here is not necessarily; closing the cross-country income gap is not an impossible task. The framework introduced in this paper reveals that productivity growth in services is positively linked to the distance to the frontier, the growth efficiency level and the investment rate of a country. If developing countries are to take seriously the task of closing the income gap, a special focus should be made to increase both investment rates in services and the growth efficiency level. Further research should be done to understand the components of this growth efficiency level.

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## A Appendix

### A.1 Fact 1 (Robustness): Variance in Services Productivity Growth is Higher in Developing Countries Than Advanced Ones

|                             | mean | variance | N  |
|-----------------------------|------|----------|----|
| <b>Advanced Economies</b>   | 1.23 | 0.91     | 4  |
| <b>Developing Economies</b> | 1.31 | 3.89     | 45 |

Table 11: Mean and Variance of Services Productivity Growth

*Notes.* The mean and variance are computed using the mean year-on-year services labour productivity growth for each country between 1990 to 2018.  $N$  represents the number of advanced and developing countries in the cross-section. The distinction between advanced and developing countries is made following the IMF's classification system.

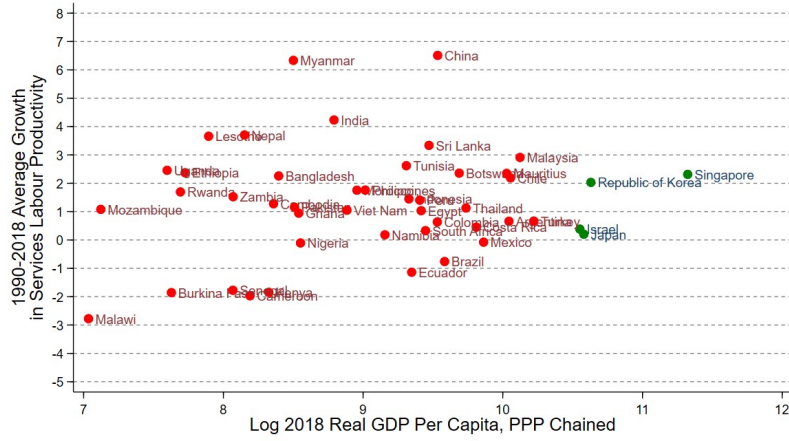


Figure 6: Average Services Productivity Growth by GDP per Capita

*Notes.* The y-axis is mean services labour productivity between 1990 and 2018 while the x-axis is the logarithm of real GDP per capita in 2018. Red represents developing countries while green represents advanced countries. The distinction between advanced and developing countries is made following the IMF's classification system.

### A.2 Fact 2 (Robustness): Services Productivity Growth is Driven by the 'Within' Channel

|                | mean | SD   | N  |
|----------------|------|------|----|
| <b>Between</b> | 0.14 | 0.34 | 42 |
| <b>Within</b>  | 0.84 | 0.32 | 42 |

Table 12: Statistics on the Decomposition of Productivity Increase

*Notes.* This table presents the mean and standard deviations of the share of the services labour productivity increase between 1975 and 2009 driven by the 'between' and 'within' forces. Namibia, Nigeria, Rwanda and Mexico are excluded due to excessively large positive and negative shares.  $N$  represents the number of advanced and developing countries in the cross-section.

### A.3 Fact 3 (Robustness): The 'Within' Channel's Effect is Partly Driven by Conditional Convergence

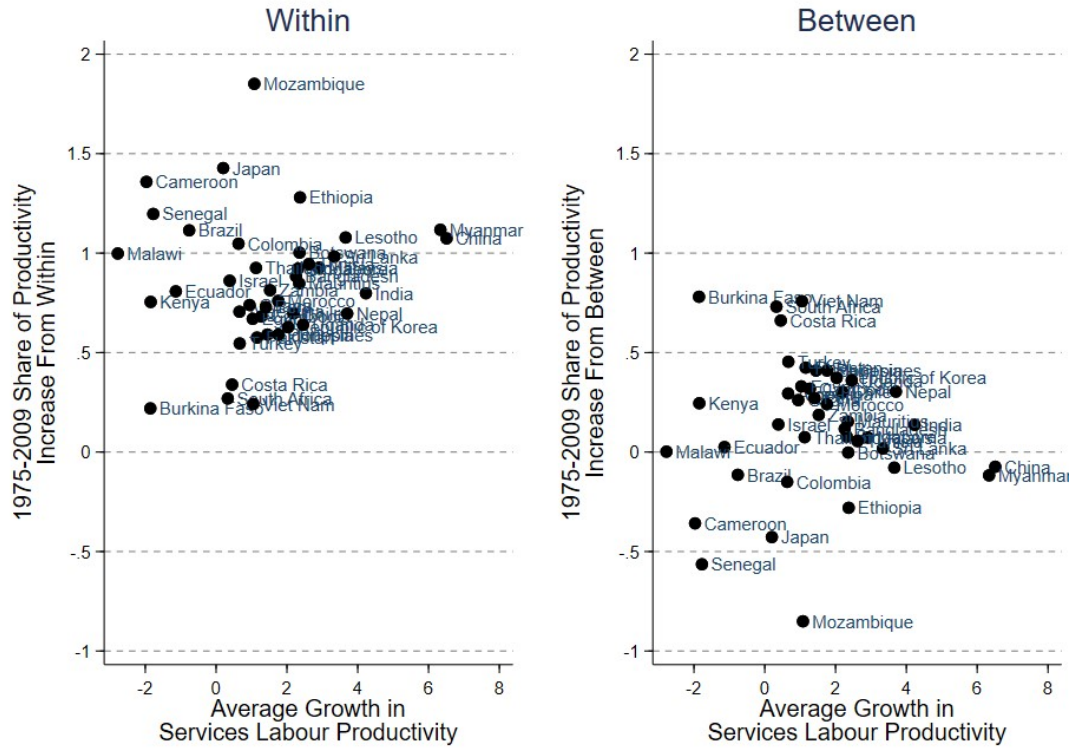


Figure 7: Statistics on the Decomposition of Productivity Increase

*Notes.* This figure plots the results of the statistical decomposition of services labour productivity growth between 1990 and 2018. The y-axis the share of the increase in services labour productivity between 1990 and 2018 driven by the ‘within’ (LHS) and ‘between’ components (RHS). The x-axis is mean year-on-year growth in services labour productivity between 1990 and 2018. Namibia, Nigeria, Rwanda and Mexico are excluded due to excessively large positive and negative shares. The interaction term is not included because it plays a minor role in this exercise.

|                  | (1)                | (2)              | (3)              |
|------------------|--------------------|------------------|------------------|
| $\ln y$          | -0.26**<br>(-2.24) | -0.09<br>(-1.09) | -0.12<br>(-0.81) |
| Industry-Year FE | ×                  | ✓                | ×                |
| Industry FE      | ✓                  | ✓                | ✓                |
| Year FE          | ×                  | ✓                | ×                |
| Country FE       | ×                  | ×                | ×                |
| N                | 190                | 570              | 190              |

Table 13: Results of Unconditional Convergence Regressions

*Notes.* This table presents the cross-section regression results of equation (3) using robust regression methods. Column (1) represents the cross-section specification, column (2) represents the panel specification and column (3) represents the specification keeping the final decade. T-statistics are in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

|                  | (1)                 | (2)                 | (3)                 |
|------------------|---------------------|---------------------|---------------------|
| $\ln y$          | -1.33***<br>(-8.87) | -1.05***<br>(-7.04) | -1.45***<br>(-6.43) |
| Industry-Year FE | ×                   | ✓                   | ×                   |
| Industry FE      | ✓                   | ✓                   | ✓                   |
| Year FE          | ×                   | ✓                   | ×                   |
| Country FE       | ✓                   | ✓                   | ✓                   |
| N                | 190                 | 570                 | 190                 |

Table 14: Results of Conditional Convergence Regressions

*Notes.* This table presents the cross-section regression results of equation (4) using robust regression methods. Column (1) represents the cross-section specification, column (2) represents the panel specification and column (3) represents the specification keeping the final decade. T-statistics are in parentheses.\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$ .

## B Model

### B.1 Proof of Proposition 1

This sub-section presents the proof of Proposition 1. Recognise that the solution to solve the intertemporal problem requires optimal control theory. Start with the present-value Hamiltonian:

$$\hat{H} = e^{-\rho t} \log c_t + \mu_t [(a_t - c_t) \Gamma(\omega, a, t) - \delta a_t]$$

where the rearranged flow budget constraint and the production function have been substituted in. The corresponding first order conditions are:

$$\hat{H}_c : \frac{e^{-\rho t}}{c_t} - \Gamma(\omega, t, a_t) \mu_t = 0$$

$$\hat{H}_a : \mu_t [\Gamma(\omega, a, t) - (a_t - c_t) \omega_t \gamma - \delta] = -\dot{\mu}_t$$

$$\hat{H}_\mu : \dot{a}_t = (y_t - c_t) \Gamma(\omega, t, a) - \delta a_t$$

$$\lim_{t \rightarrow \infty} \mu_t a_t = 0$$

Take  $\hat{H}_c$  and manipulate:

$$\frac{e^{-\rho t}}{c_t} - \Gamma(\omega, t, a) \mu_t = 0$$

$$\frac{e^{-\rho t}}{c_t} = \Gamma(\omega, t, a) \mu_t$$

$$c_t = \frac{e^{-\rho t}}{\Gamma(\omega, t, a) \mu_t}$$

Note that  $\mu_t$  is the shadow value of an extra unit of wealth. This changes depending on whether the planner chooses catch-up (transitional dynamics) or CRS technology (balanced growth path). The Euler equation will therefore change according to the technology choice. Take logs of  $c_t$ :

$$\log(c_t) = -\rho t \log(e) - [\log(\mu_t) + \log(\Gamma(\omega, t, a))]$$

Acknowledge that in ‘catch-up phase’,  $\omega_t = 1$  so replace  $\Gamma(\omega, t, a)$  with  $\gamma(\bar{a}_t - a_t)$ :

$$\log(c_t) = -\rho t \log(e) - [\log(\mu_t) + \log(\gamma) + \log(\bar{a}_t - a_t)]$$

$$\log(c_t) = -\rho t \log(e) - \log(\mu_t) - \log(\gamma) - \log(\bar{a}_t - a_t)$$

Now take the time derivative of this:

$$\frac{\dot{c}_t}{c_t} = -\rho - \frac{\dot{\mu}_t}{\mu_t} - \frac{(\dot{\bar{a}}_t - \dot{a}_t)}{(\bar{a}_t - a_t)}$$

Substitute in the LOM for domestic technology:

$$\frac{\dot{c}_t}{c_t} = -\rho - \frac{\dot{\mu}_t}{\mu_t} - \frac{(\bar{a}_t g_{\bar{a}} - \gamma(\bar{a}_t - a_t)z_t + \delta a_t)}{(\bar{a}_t - a_t)}$$

What remains is obtaining an expression for  $-\frac{\dot{\mu}_t}{\mu_t}$ . To do this, take  $\hat{H}_a$  and manipulate:

$$\mu_t[\Gamma(\omega, a, t) - (a_t - c_t)\omega_t\gamma - \delta] = -\dot{\mu}_t$$

$$\Gamma(\omega, a, t) - (a_t - c_t)\omega_t\gamma - \delta = \frac{-\dot{\mu}_t}{\mu_t}$$

Furthermore,  $\Gamma(\omega, a, t) = \gamma(\bar{a}_t - a_t)$  as  $\omega_t = 1$ :

$$\frac{-\dot{\mu}_t}{\mu_t} = \gamma(\bar{a}_t - a_t) - (a_t - c_t)\gamma - \delta$$

Restating the Euler quation:

$$\frac{\dot{c}_t}{c_t} = -\rho - \frac{(\bar{a}_t g_{\bar{a}} - \gamma(\bar{a}_t - a_t)z_t + \delta a_t)}{(\bar{a}_t - a_t)} - \frac{\dot{\mu}_t}{\mu_t}$$

Where:

$$\frac{-\dot{\mu}_t}{\mu_t} = \gamma(\bar{a}_t - a_t) - (a_t - c_t)\gamma - \delta$$

To get the equilibrium law of motion for catch-up investment, recognise that  $\omega_t = 1$ :

$$\dot{a}_t = (a_t - c_t)\Gamma(\omega, a, t) - \delta a_t = z_t(\bar{a}_t - a_t)\gamma - \delta a_t$$

## B.2 Proof of Proposition 2

This sub-section presents the proof of Proposition 2. The Hamiltonian and the corresponding first order conditions are unchanged from Proposition 1:

$$c_t = \frac{e^{-\rho t}}{\Gamma(\omega, t, a)\mu_t}$$

Take logs of  $c_t$  and take the time derivative as before:

$$\log(c_t) = -\rho t \log(e) - [\log(\mu_t) + \log(\Gamma(\omega, t, a))]$$

In the Balanced Growth Path,  $\omega_t = 0$  so  $\Gamma(\delta, a, t) = \eta$ :

$$\log(c_t) = -\rho t \log(e) - [\log(\mu_t) + \log(\eta)]$$

Take the time derivative:

$$\frac{\dot{c}_t}{c_t} = -\rho - \frac{\dot{\mu}_t}{\mu_t}$$

What remains is obtaining an expression for  $-\frac{\dot{\mu}_t}{\mu_t}$ . To do this, take  $\hat{H}_a$  and manipulate:

$$\mu_t[\Gamma(\omega, a, t) - (a_t - c_t)\omega_t\gamma - \delta] = -\dot{\mu}_t$$

$$\Gamma(\omega, a, t) - (a_t - c_t)\omega_t\gamma - \delta = \frac{-\dot{\mu}_t}{\mu_t}$$

Since CRS technology is preferred,  $\omega_t = 0$  so  $\Gamma(\omega, a, t) = \eta$ :

$$\frac{-\dot{\mu}_t}{\mu_t} = \eta - \delta$$

Upon substitution, the Euler equation is:

$$\frac{\dot{c}_t}{c_t} = \eta - \delta - \rho$$

To get the equilibrium law of motion for innovation investment, recognise that  $\omega_t = 0$  which means:

$$\dot{a}_t = (a_t - c_t)\Gamma(\omega, a, t) - \delta a_t = z_t\eta - \delta a_t$$